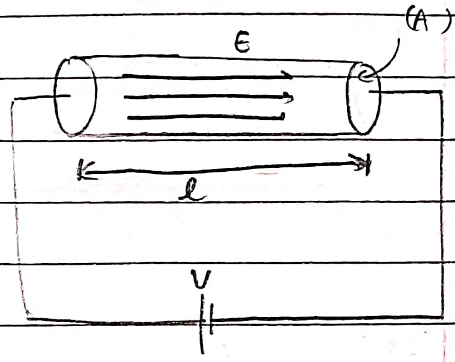
CURRENT ELECTRICITY

/06/2023

Considering homogenous conductor



$E = \frac{V}{l} \Rightarrow F_e = e\left(\frac{V}{l}\right)$

$\Rightarrow a_e = \left(\frac{e}{m}\right)\left(\frac{V}{l}\right)$

Let τ be the time b/w successive collisions of e^-s.
Assuming e start from rest after every collision.

$v = a\tau = \left(\frac{e}{m}\right)\left(\frac{V}{l}\right)(\tau)$

$V_{avg} = \frac{0+v}{2}$ ($\because$ uniform acc.)

$= v/2$

$V_{drift} = V_{avg} + V_{\text{due to random motion}} = \frac{v}{2} + \frac{v}{2}$ (somehow)

$= v$

- Drift speed/vel. — Speed with which e^- move in opp. dirxn to E applied.

NOTE: $v_d \neq$ Speed of current

When switch closed, current flows instantaneously

Let n be the # free e^-s.

In time dt, $dq = \overbrace{ne}^{\text{(Charge density)}} \overbrace{(A v_d\, dt)}^{\text{(Vol.)}}$ pass through a certain cross section

$$i = \frac{dq}{dt} \Rightarrow \boxed{i = neAv_d}$$

$$i = neA\left(\frac{e}{m}\right)(\tau)\left(\frac{V}{l}\right) = \left(\frac{ne^2\tau}{m}\right)\left(\frac{VA}{l}\right)$$

(Depends on material) (Depends on Temp.) (Depend on Geometry of cond.)

$$R = \frac{V}{i} = \left(\frac{m}{ne^2\tau}\right)\left(\frac{l}{A}\right)$$

$$= \rho\frac{l}{A}$$

(Resistivity or specific resistance)

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

(Conductivity) (Conductivity)

$$\boxed{\sigma = \frac{1}{\rho}} \qquad \boxed{G = \frac{1}{R}}$$

Unit: Mho (Ω^{-1})

** $\frac{\rho l}{GM(A_1, A_2)}$

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$$\frac{i}{A} = \sigma E \Rightarrow \boxed{\vec{j} = \sigma \vec{E}}$$

(current density)

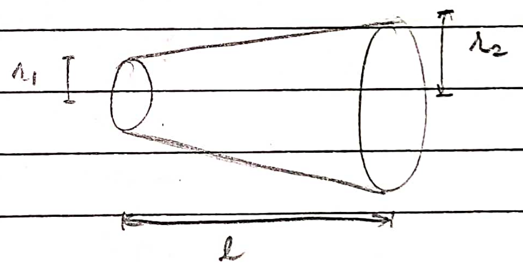
* Normal to cross-sectional area

• Thermal/ Temp. coeff. of Resistance (α) — $\boxed{R = R_0(1 + \alpha \Delta T)}$

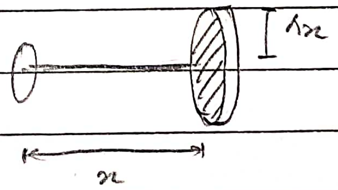
NOTE: $\alpha > 0$ for conductors

$\alpha < 0$ for semi-conductors

Q. Find R



A. $dR = \frac{\rho\, dx}{A_x}$



$$= \frac{\rho l^2\, dx}{\pi^2 (l r_1 + (r_2 - r_1)x)^2}$$

$$r_x = r_1 + \frac{(r_2 - r_1)\, x}{l}$$

All elems. in series.

$$= \frac{(l - x) r_1 + x r_2}{l}$$

$$\Rightarrow R = \int dR = \int_0^l \frac{\rho l^2\, dx}{\pi^2 (l r_1 + (r_2 - r_1)x)^2}$$

$$= \left(\frac{\rho l^2}{\pi^2}\right)\left(\frac{1}{r_2 - r_1}\right)\left[\frac{1}{l r_1} - \frac{1}{l r_2}\right] = \frac{\rho l}{\pi r_1 r_2} \;**$$

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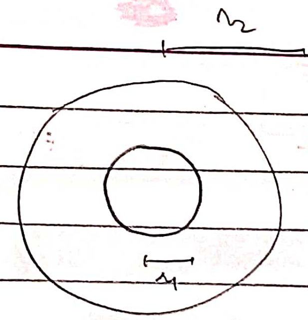
Q. 

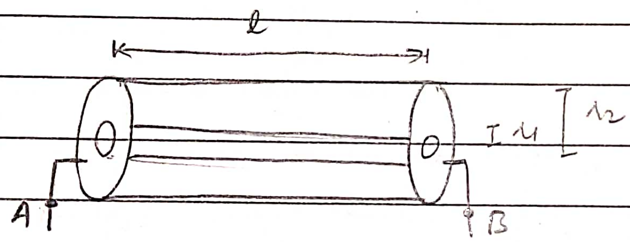
Find R.

(All elems in series)

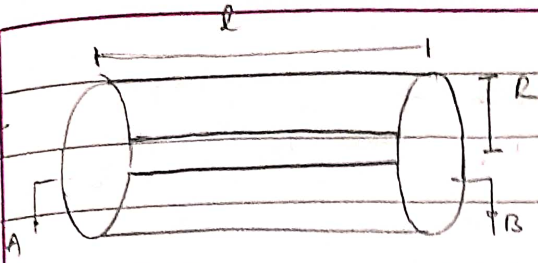
A. $dR = \frac{\rho\, dx}{A_x} = \frac{\rho\, dx}{4\pi x^2} \Rightarrow R = \int dR$

$$= \int_{r_1}^{r_2} \frac{\rho\, dx}{4\pi x^2}$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Q. 

Find R.



Q. Find R if $\rho = \rho_0\left(1+\frac{r}{R}\right)$

$r \to$ dist from axis of cyl.

A. $dR = \frac{\rho_x l}{A_x} = \rho_0\left(\frac{1+x}{R}\right)\left(\frac{l}{2\pi x\, dx}\right)$

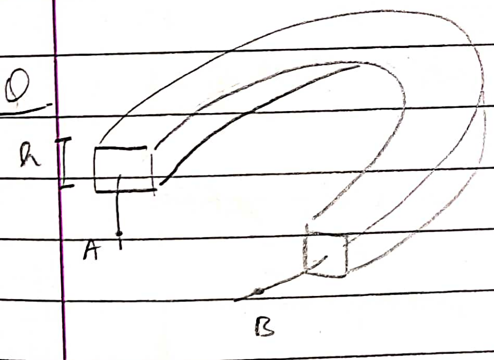
All elems in $\parallel$ $\Rightarrow$ $\frac{1}{dR} = \frac{2\pi R}{\rho_0 l}\left(\frac{x}{R+x}\right) dx$

$$\frac{1}{R} = \int \frac{1}{dR} = \int_0^R \left(\frac{2\pi R}{\rho_0 l}\right)\left(1 - \frac{R}{R+x}\right) dx$$

$$= \frac{2\pi R^2}{\rho_0 l}\left[1 - \ell(2)\right]$$

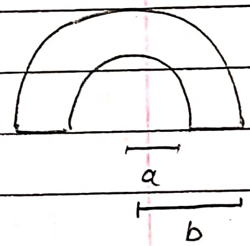
$$\Rightarrow \quad R = \frac{\rho_0 l}{2\pi R^2 (1-\ell(2))}$$

Q.

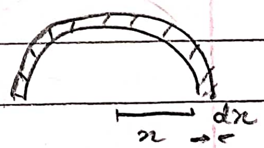


Semicircular

Find R.



A. $dR = \frac{\rho(\pi x)}{h\, dx}$

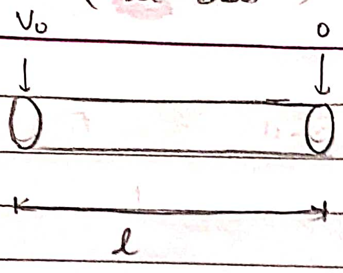


$$\frac{1}{R} = \int \frac{1}{dR} = \int_a^b \frac{h}{\rho\pi}\frac{dx}{x}$$

$$= \frac{h}{\rho\pi}\,\ell\left(\frac{b}{a}\right) \Rightarrow R = \frac{\rho\pi}{h\,\ell(b/a)}$$

(V maintained at ends)

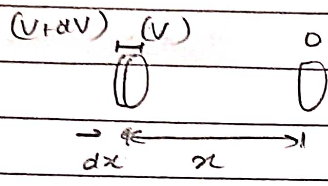
Q. (A)



$$\rho = \rho_0\left(1+\frac{V}{V_0}\right)$$

find R.

A.



$$(V+dV) - V = i\,dR \quad \left(i \text{ same through all cross sections}\right)$$

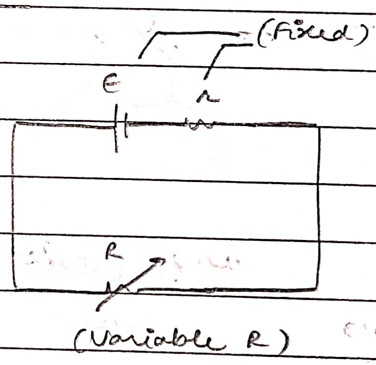
$$\Rightarrow dV = \frac{i\,dx}{A}\rho_0\left(1+\frac{V}{V_0}\right)$$

$$\Rightarrow \int_0^{V_0}\frac{dV}{V+V_0} = \int_0^{l}\frac{\rho_0 i}{V_0 A}\,dx$$

$$\Rightarrow \ln(2) = \frac{\rho_0 i l}{V_0 A}$$

$$\Rightarrow i = \frac{V_0 A}{\rho l}\ln(2) \quad \Rightarrow \quad R = \frac{V}{i} = \frac{\rho l}{A\ln(2)}$$

→ Max power transfer theorem



$$P = i^2 R$$

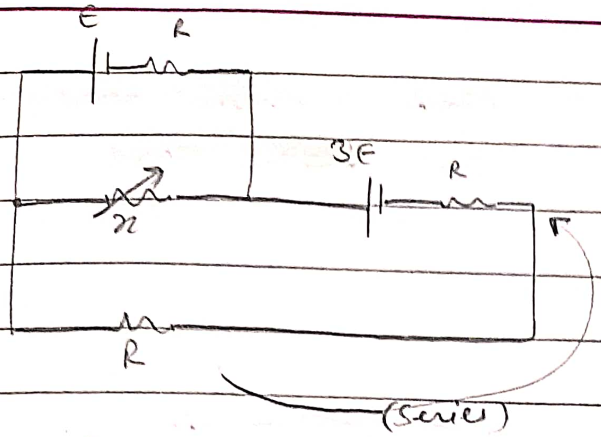
$$= \frac{E^2 R}{(R+r)^2}$$

$$\frac{dP}{dR} = E^2\,\frac{(R+r)^2 - 2R(R+r)}{(R+r)^2}$$

$$= E^2\,\frac{(r^2-R^2)}{(R+r)^2} = 0 \Rightarrow R = r, -r$$

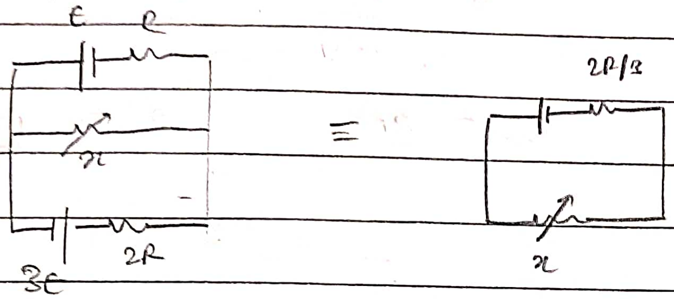
$P = P_{max}$ for $\boxed{R = r}$

Q.



Find x for which power consumed in the resistor is max.

A



$\Rightarrow P = P_{max}$ for $x = 2R/3$

→ Heating effect of current (Joule's law of heating)

Power consumed by a resistor is dissipated as heat.

$$\boxed{H = i^2Rt = \frac{V^2 t}{R} = iVt}$$

For any load, marked voltage & marked power is given.

eg 100 W at 220 V.

$\Downarrow$

Load consumes 100W at 220V.

$$\Rightarrow R_{load} = \frac{(220)^2}{100} = 484\,\Omega$$

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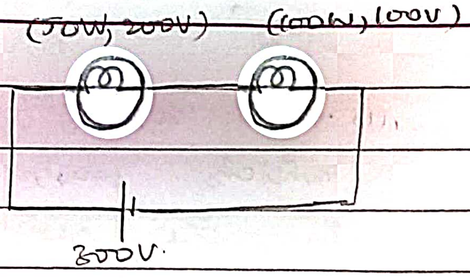
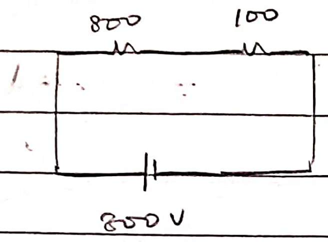
Q. 

Find power consumed by each load.

A. 

$i = \frac{300}{900} = \underline{1/3}$

$P_{800} = \frac{800}{9}$ $\quad P_{100} = \frac{100}{9}$

08/06/2023

Q. The current through resistor of resistance R is varied from 0 to i_0 linearly with time from $t=0$ to $t=T$. Find heat produced.

A. $i = \frac{i_0}{T} t$ $\quad dH = i^2 R\, dt = \frac{i_0^2}{T^2} R\, t^2\, dt$

$H = \frac{i_0^2}{3T^2} R (T^3) = \frac{i_0^2 RT}{3}$

Q. Total charge q is passed through resistor of resistance R in a time interval of $t=0$ to $t=T$. The current in resistor decreases linearly with time with some initial value at $t=0$ to zero at $t=T$.

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A. $i = i_0\left(1 - \frac{t}{T}\right)$

$dq = i\,dt$

$= i_0\left(1 - \frac{t}{T}\right) dT$

$q = i_0\left(T - \frac{T^2}{2T}\right)$

$= \left(\frac{i_0 T}{2}\right)$

$dH = i^2 R\,dt$

$= i_0^2 R\left(1 - \frac{2t}{T} + \frac{t^2}{T^2}\right) dt$

$H = i_0^2 R\left(T - \frac{T^2}{T} + \frac{T^3}{3T^2}\right)$

$= \frac{i_0^2 R T}{3} = \frac{4q^2 R}{3T}$

Q. In the above Q, if i_0 halves continuously after T time, find heat produced.

A. $i = i_0\, e^{-\frac{\ell(2) t}{T}}$

$dq = i_0\, e^{-\frac{\ell(2)}{T} t}\, dt$

$q = -\frac{i_0 T}{\ell(2)}\left[e^{-\frac{\ell(2)}{T} t}\right]_0^{\infty}$

$= \frac{i_0 T}{\ell(2)}$

$dH = i^2 R\, dt$

$= i_0^2 R\, e^{-\frac{2\ell(2)}{T} t}\, dt$

$\Rightarrow H = -i_0^2 R\left(\frac{T}{2\ell(2)}\right)\left[e^{-\frac{2\ell(2)}{T} t}\right]_0^{\infty}$

$= \frac{i_0^2 R T}{2\ell(2)} = \left(\frac{q^2}{T^2}\right)\frac{\ell^2(2) R T}{2\ell(2)}$

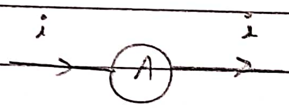
$= \frac{q^2 R\,\ell(2)}{2T}$

ELECTRICAL INSTRUMENTS

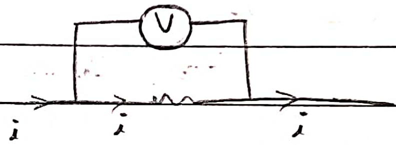
→ **Ammeter & Voltmeter**

Ideal (A) — Zero resistance

Ideal (V) — ∞ resistance



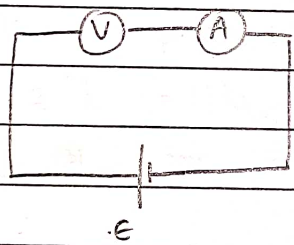
(Attached in Series)



(Attached in parallel)

NOTE: Ammeter measures the current passing through it.

Q. If

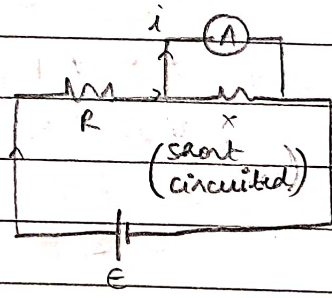


Reading

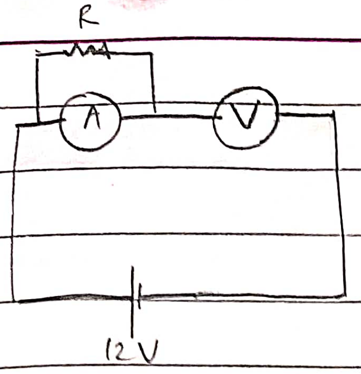
A — 0 Amp.

V — E Volt

Q. Find reading of (A)



A. $i = \left(\frac{E}{R}\right)$



Q. After connecting R

Readings

A: $i_0 \rightarrow i_0/2$

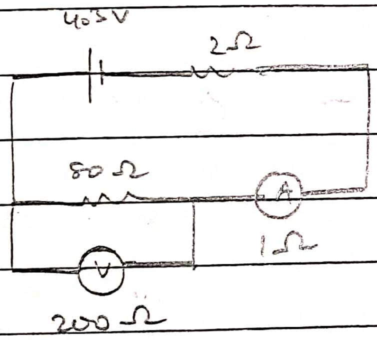
V: $V_0 \rightarrow 2V_0$

Find V_0

A. (ML)

1. $i_0 R_A + V_0 = 12$ ← (Before)
2. $\frac{i_0}{2} R_A + 2V_0 = 12$ ← (After)

$\Rightarrow V_0 = 4$



Q. Find readings of A & V

A. $R_{eq} = 2 + 1 + \frac{50 \times 200}{250} = 43 \Rightarrow i = 0.1$

A: 0.1 Amp

V: $4.3 - (0.1)(1+2) = 4$ volt

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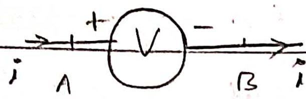
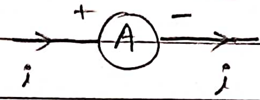
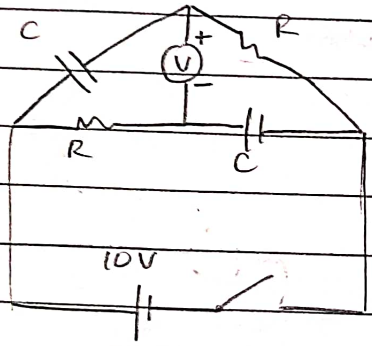
- **Polarity** –





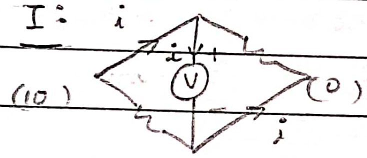
$$V_A - V = V_B$$
$$\Rightarrow \underline{V_A - V_B = V}$$

Q.



Caps initially uncharged & switch closed. Find reading of (V) in steady – state. Also find time after which reading is zero.

A. I: i



$$\underline{V_i = 10}$$

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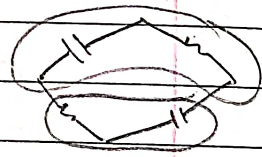
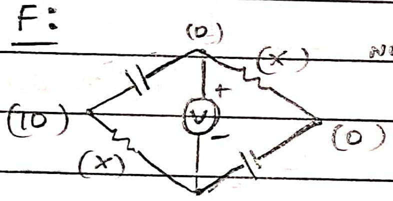


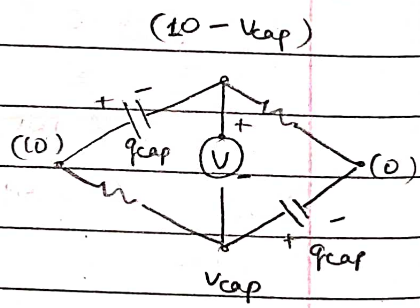
F:



no current

$$\underline{V_f = -10}$$

{ These are 2 independent circuits. Hence we are able to apply RC-circuit formulae. }



At time 't', $q_{cap} = \epsilon C(1 - e^{-\frac{t}{RC}})$

$$V_{cap} = \frac{q_{cap}}{C} = \epsilon(1 - e^{-\frac{t}{RC}})$$

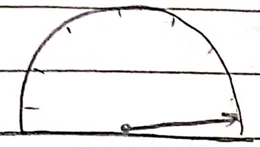
$$V = (10 - V_{cap}) - V_{cap}$$
$$= 10 - 2\epsilon(1 - e^{-t/RC})$$
$$= 20e^{-t/RC} - 10$$

for $V = 0$

$$\Rightarrow \underline{t = RC\,\ell(2)}$$

• Conversion from Galvanometer –

Galvanometer – Device which measures current.

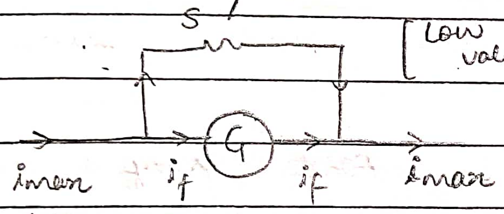


Full scale deflection current (i_f)

$= \left(\text{Max current measured by } G \right)$

(Shunt resistance)

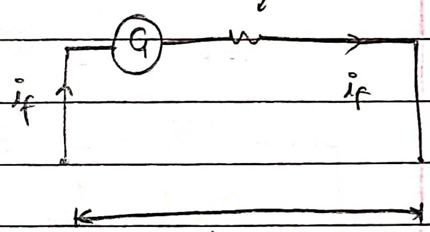
[Low in value]



(Max i measured by A)

Ammeter

(High resistance)



Voltmeter

(Max voltage measured by V)

$S(i_{max} - i_f) = G i_f$

$\Rightarrow \boxed{S = \dfrac{G i_f}{(i_{max} - i_f)}}$

$i_f (G + R_H) = V_{max}$

$\boxed{R_H = \dfrac{V_{max}}{i_f} - G}$

• We choose S & R_H in such a manner so as to ensure only i_f passes through G

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Q $G = 25\,\Omega$ & $i_f = 10^{-3}$ A

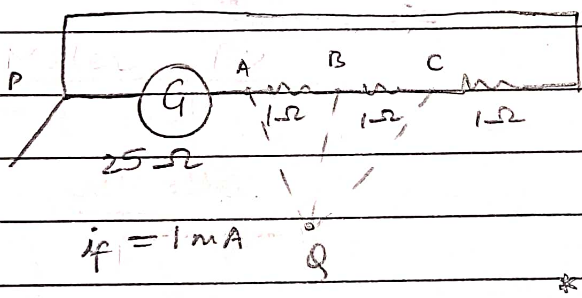
find a) S for (A) with $i_{max} = 10$ A

b) R_H for (V) with $V_{max} = 100$ V

A 1. $S = \dfrac{(25)(10^{-3})}{(10 - 10^{-3})} \sim \underline{2.5 \times 10^{-3}\,\Omega}$

2. $R_H = \dfrac{100}{10^{-3}} - 100 \sim \underline{10^5\,\Omega}$

Q



Find ranges of (A) when Q attached with A, B, C.

*(Resistance of G & those in series with G)

A. $i_{max} = \dfrac{G\, i_f}{S} + i_f$

1. $i_{max} = \dfrac{(25)(10^{-3})}{3} + 10^{-3} = \underline{9.3 \times 10^{-3}}$

2. $i_{max} = \dfrac{(25+1)(10^{-3})}{2} + 10^{-3} = \underline{1.4 \times 10^{-2}}$

3. $i_{max} = \dfrac{(25+2)(10^{-3})}{1} + 10^{-3} = \underline{2.8 \times 10^{-2}}$

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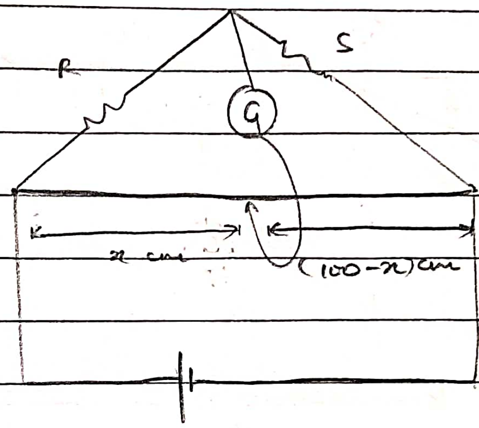
→ Meter Bridge & Post Office Box

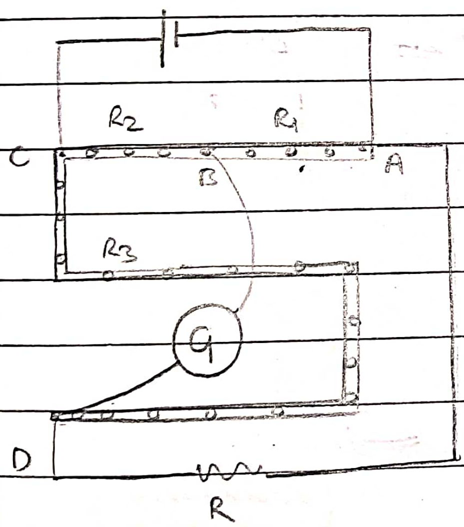
Principle : Wheatstone Bridge

$$\boxed{\frac{R}{S} = \left(\frac{x}{100-x}\right)}$$

when deflection in G is zero



Meter Bridge



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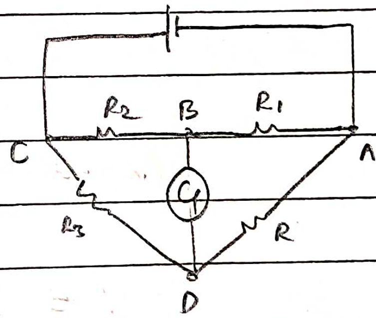
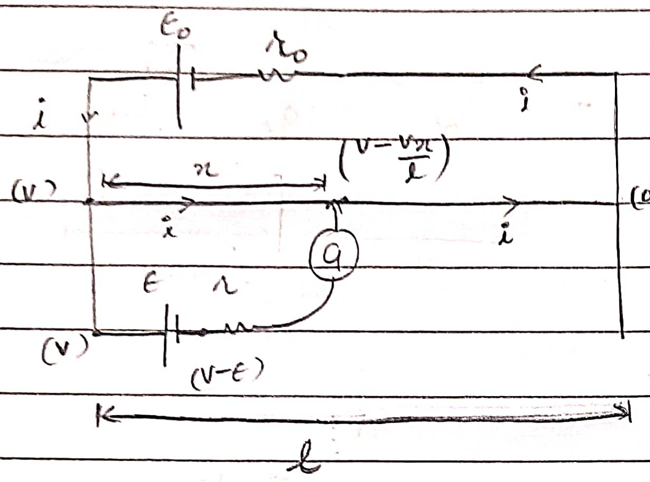


$$\boxed{\frac{R_3}{R_2} = \frac{R}{R_1}}$$

when deflection in G is zero.

→ Potentiometer

used to find EMF & internal resistance of battery.



When deflection in G is zero. no current through r.

$$\Rightarrow V - E = V - \frac{Vx}{l}$$

$$\Rightarrow \boxed{E = \left(\frac{Vx}{l}\right)}$$

For comparing EMFs of two batteries, $\boxed{\frac{E_1}{E_2} = \frac{x_1}{x_2}}$

Here, Potential gradient $= \left(\frac{V}{l}\right)$

& $V = \left(\frac{E_0}{R + r_0}\right)(R)$ ← (resistance of wire)

Q. If post of no deflection can't be obtained, what could be the error?

A. 1) Polarity of batteries may be opposite.

2) $E > E_0$

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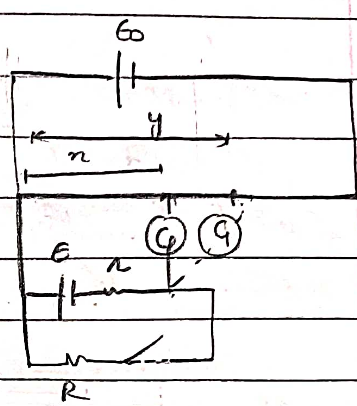
To find r, we connect a known resistance R in $\parallel$ to r.

We measure posts. of no deflection before & after closing the switch



$$\epsilon = \frac{Vx}{l}$$

$$\epsilon - ir = \frac{V}{l}y \quad ; \quad i = \frac{\epsilon}{(R+r)}$$

$$\Rightarrow \left(\frac{\epsilon R}{R+r}\right) = \frac{V}{l}y$$

$$\Rightarrow xR = y(R+r) \Rightarrow \boxed{r = \left(\frac{x-y}{y}\right)R}$$

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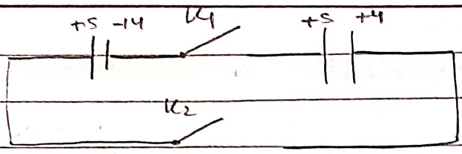
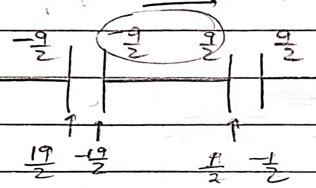
☆ Q. 

Find charge distribution when K_1 closed.

A. circuit not complete, so charge shouldn't flow.

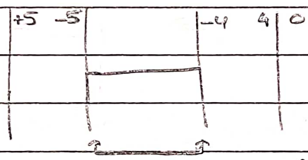
☆ But, this happens when only insides of the plates are charged



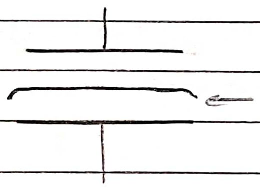
⟹ Field created by these charges
⟹ Charge starts flowing.

⟹ Charge distr of 4 plates

$\left(\frac{5-14+5+4}{2}\right) = 0$



* These are technically one plate since no charge left on outside plates ⟹ No E ⟹ same V

☆ Q. 

To what height does water rise if σ_0 charge density on plates of cap.

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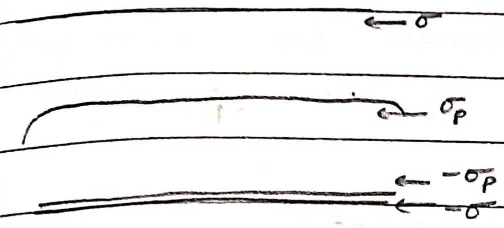
A. Let charge density be σ at a certain inst.



$$dF = q_p\, dE \quad \left(\text{E due to lower plate}\right)$$

$$dF = (\sigma_p A)\left(\frac{\sigma+d\sigma}{2\epsilon_0} + \frac{\sigma+d\sigma}{2\epsilon_0} + \frac{\sigma_p}{2\epsilon_0}\right)$$

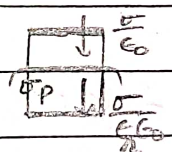
(E due to upper plate) (E due to lower water surface)

After time dt, it changes to $(\sigma + d\sigma)$, σ_p remaining const.

$$-(\sigma_p A)\left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} + \frac{\sigma_p}{2\epsilon_0}\right)$$

$$\Rightarrow dF = \frac{\sigma_p A\, d\sigma}{\epsilon_0}$$

By GL.



$$\sigma\left(\frac{1}{\epsilon_0} - \frac{1}{\epsilon_r\epsilon_0}\right)A = \frac{\sigma_p A}{\epsilon_0}$$

$$\Rightarrow \sigma_p = \sigma\left(1 - \frac{1}{\epsilon_r}\right)$$

$$= \frac{A}{\epsilon_0}\left(1 - \frac{1}{\epsilon_r}\right)\sigma\, d\sigma$$

$$\Rightarrow F = \frac{A}{2\epsilon_0}\left(1 - \frac{1}{\epsilon_r}\right)\sigma_0^2$$

This force will balance the wt. of risen water,

$$\Rightarrow F = mg = \rho g h A$$

$$\Rightarrow \boxed{h = \frac{(\epsilon_r - 1)\sigma^2}{2\epsilon_r\epsilon_0 \rho g}}$$

* 

When switch closed, current experiences impulsive force ⇒ Energy conservation cannot be used.

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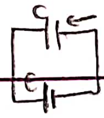
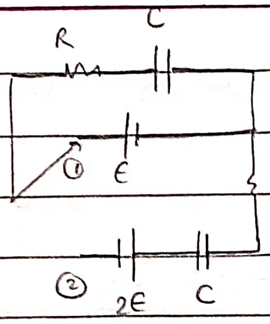
eg: 

$q = C\epsilon$ $W_{battery} = qV = C\epsilon^2$

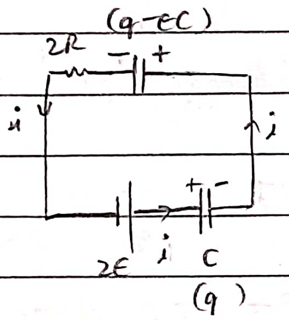
$U_{cap} = \frac{1}{2}C\epsilon^2$

$\Rightarrow \boxed{W_{battery} \neq U_{cap}}$

⇒ Energy lost as heat.

Q. 

Initially steady state. At $t=0$, switch shifted from ① to ②. Find charge on uncharged cap. as a fn of time.

A. ≡ 

$i = \frac{dq}{dt}$

(KL) $2\epsilon - \frac{q}{C} - \left(\frac{q-\epsilon C}{C}\right) - 2Ri = 0$

$\Rightarrow 2\epsilon - \frac{2q}{C} + \epsilon = 2R\frac{dq}{dt}$

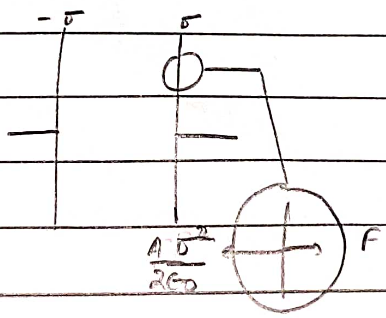
$\Rightarrow 2R\frac{dq}{dt} + \frac{2q}{C} = 3\epsilon$

$\Rightarrow q = (3\epsilon)\left(\frac{C}{2}\right)\left(1 - e^{-\frac{t}{RC}}\right)$

*Q. Each plate of a parallel plate capacitor has area 'A'. Find amt. of work done to ↑ the distance of plates from x_1 to x_2, keeping voltage across capacitor to be const. 'V'.

A. * Whenever battery attached, Energy conservation cannot be used.

So we need to used force.



For moving slowly, $F = \frac{A\sigma^2}{2\epsilon_0} = \frac{V^2 A\epsilon_0}{2x^2}$

$V = \frac{\sigma x}{\epsilon_0}$

$dW = F\cdot dx = \frac{V^2 A\epsilon_0}{2}\frac{dx}{x^2}$

$W = \int_{x_1}^{x_2} dW = \boxed{\left(\frac{V^2 A\epsilon_0}{2}\right)\left(\frac{1}{x_1} - \frac{1}{x_2}\right)}$